

## Chapter 7

## Exercise 7A

1. (a)  $\mathbf{r}_A = 2\mathbf{i} + 3\mathbf{j}$   
 (b)  $\mathbf{r}_B = 4\mathbf{i} + 5\mathbf{j}$   
 (c)  $\mathbf{r}_C = \mathbf{i} + 4\mathbf{j}$   
 (d)  ${}_A\mathbf{r}_B = \mathbf{r}_B - \mathbf{r}_A = -2\mathbf{i} - 2\mathbf{j}$   
 (e)  ${}_A\mathbf{r}_C = \mathbf{r}_C - \mathbf{r}_A = \mathbf{i} - \mathbf{j}$   
 (f)  ${}_A\mathbf{r}_D = \mathbf{r}_D - \mathbf{r}_A = -2\mathbf{i} + 2\mathbf{j}$   
 (g)  ${}_D\mathbf{r}_A = \mathbf{r}_A - \mathbf{r}_D = 2\mathbf{i} - 2\mathbf{j}$   
 (h)  ${}_B\mathbf{r}_C = \mathbf{r}_C - \mathbf{r}_B = 3\mathbf{i} + \mathbf{j}$   
 (i)  ${}_C\mathbf{r}_D = \mathbf{r}_D - \mathbf{r}_C = -3\mathbf{i} + 3\mathbf{j}$   
 (j)  ${}_D\mathbf{r}_C = \mathbf{r}_C - \mathbf{r}_D = 3\mathbf{i} - 3\mathbf{j}$
2. (a)  $\mathbf{r}_A$  is given  
 (b)  $\mathbf{r}_B$  is given  
 (c)  ${}_C\mathbf{r}_A = \mathbf{r}_C - \mathbf{r}_A$  so  
 $\mathbf{r}_C = {}_C\mathbf{r}_A + \mathbf{r}_A = 2\mathbf{i} + \mathbf{j}$   
 (d)  $\mathbf{r}_D = {}_D\mathbf{r}_A + \mathbf{r}_A = \mathbf{i} + 3\mathbf{j}$   
 (e)  $\mathbf{r}_E$  is given  
 (f)  $\mathbf{r}_F = {}_F\mathbf{r}_E + \mathbf{r}_E = -\mathbf{i} + 5\mathbf{j}$   
 (g)  $\mathbf{r}_G = {}_G\mathbf{r}_D + \mathbf{r}_D = -3\mathbf{i} + 2\mathbf{j}$   
 (h)  $\mathbf{r}_H = {}_H\mathbf{r}_G + \mathbf{r}_G = -2\mathbf{i} + \mathbf{j}$   
 (i)  $\mathbf{r}_I = {}_I\mathbf{r}_A + \mathbf{r}_A = 5\mathbf{i} + 5\mathbf{j}$   
 (j)  ${}_I\mathbf{r}_J = \mathbf{r}_I - \mathbf{r}_J$  so  
 $\mathbf{r}_J = \mathbf{r}_I - {}_I\mathbf{r}_J = 5\mathbf{i} + 2\mathbf{j}$   
 (k)  $\mathbf{r}_K = \mathbf{r}_H - {}_H\mathbf{r}_K = 3\mathbf{i}$   
 (l)  $\mathbf{r}_L = \mathbf{r}_I - {}_I\mathbf{r}_L = 4\mathbf{j}$
3.  $\mathbf{r}_A = 7\mathbf{i} + 11\mathbf{j}$   
 ${}_B\mathbf{r}_A = -12\mathbf{i} - 8\mathbf{j}$   
 $\mathbf{r}_B = \mathbf{r}_A + {}_B\mathbf{r}_A = (-5\mathbf{i} + 3\mathbf{j})\text{km}.$

4.  $\overrightarrow{AB}$  represents the displacement *from A to B*, thus it is the displacement of B relative to A.

**Note the importance of understanding this result:**

$$\overrightarrow{AB} = {}_B\mathbf{r}_A$$

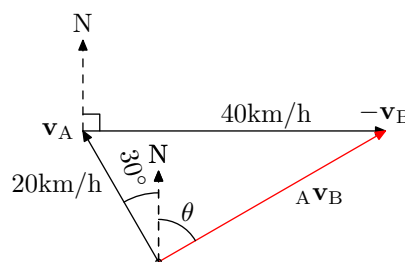
Some of you will get this the wrong way around if you are not careful!

5.  $\mathbf{r}_A = 11\mathbf{i} - 3\mathbf{j}$   
 ${}_A\mathbf{r}_B = -8\mathbf{i} - 8\mathbf{j}$   
 $\mathbf{r}_B = \mathbf{r}_A + {}_A\mathbf{r}_B = 19\mathbf{i} + 5\mathbf{j}$   
 $|\mathbf{r}_B| = \sqrt{19^2 + 5^2} = \sqrt{386}\text{km}.$
6.  $\mathbf{r}_A = 5\mathbf{i} + 2\mathbf{j}$   
 ${}_A\mathbf{r}_B = 8\mathbf{i} + 3\mathbf{j}$   
 $\mathbf{r}_B = \mathbf{r}_A + {}_A\mathbf{r}_B = -3\mathbf{i} - \mathbf{j}$   
 $|\mathbf{r}_B| = \sqrt{3^2 + 1^2} = \sqrt{10}\text{km}.$
7.  $\mathbf{r}_A = 2\mathbf{i} + 4\mathbf{j}$   
 ${}_C\mathbf{r}_A = -4\mathbf{i} - \mathbf{j}$   
 $\mathbf{r}_C = \mathbf{r}_A + {}_C\mathbf{r}_A = -2\mathbf{i} + 3\mathbf{j}$   
 $\mathbf{r}_B = \mathbf{r}_C + {}_B\mathbf{r}_C = 5\mathbf{i} - \mathbf{j}$
8. LHS:

$$\begin{aligned} {}_D\mathbf{r}_E + {}_E\mathbf{r}_F &= (\mathbf{r}_D - \mathbf{r}_E) + (\mathbf{r}_E - \mathbf{r}_F) \\ &= \mathbf{r}_D - \mathbf{r}_E + \mathbf{r}_E - \mathbf{r}_F \\ &= \mathbf{r}_D - \mathbf{r}_F \\ &= {}_D\mathbf{r}_F \\ &= \text{RHS. Q.E.D.} \end{aligned}$$

## Exercise 7B

1.  ${}_A\mathbf{v}_B = \mathbf{v}_A - \mathbf{v}_B = (2\mathbf{i} - 3\mathbf{j}) - (-4\mathbf{i} + 7\mathbf{j}) = 6\mathbf{i} - 10\mathbf{j}$
2.  ${}_A\mathbf{v}_B = (4\mathbf{i} + 2\mathbf{j}) - (7\mathbf{i} - \mathbf{j}) = -3\mathbf{i} + 3\mathbf{j}$
3.  ${}_A\mathbf{v}_B = (3\mathbf{i} - 2\mathbf{j}) - (4\mathbf{i} + 7\mathbf{j}) = -\mathbf{i} - 9\mathbf{j}$
4.  ${}_A\mathbf{v}_B = (6\mathbf{i} + 2\mathbf{j}) - (3\mathbf{i} - \mathbf{j}) = 3\mathbf{i} + 3\mathbf{j}$
5.  ${}_A\mathbf{v}_B = \mathbf{v}_A + (-\mathbf{v}_B)$



By the cosine rule,

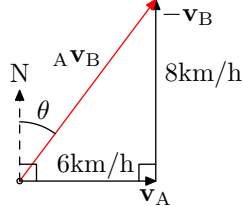
$$\begin{aligned} |{}_A\mathbf{v}_B|^2 &= 20^2 + 40^2 - 2 \times 20 \times 40 \cos 60^\circ \\ |{}_A\mathbf{v}_B| &= 20\sqrt{3} \\ &\approx 36.64\text{km/h} \end{aligned}$$

By the sine rule,

$$\begin{aligned}\frac{\sin(\theta + 30^\circ)}{40} &= \frac{\sin 60^\circ}{20\sqrt{3}} \\ \sin(\theta + 30^\circ) &= \frac{40 \times \frac{\sqrt{3}}{2}}{20\sqrt{3}} \\ &= 1 \\ \therefore \theta + 30^\circ &= 90^\circ \\ \theta &= 60^\circ\end{aligned}$$

${}_A\mathbf{v}_B = 36.64\text{km/h}$  on a bearing of  $060^\circ$ .

6.  ${}_A\mathbf{v}_B = \mathbf{v}_A + (-\mathbf{v}_B)$



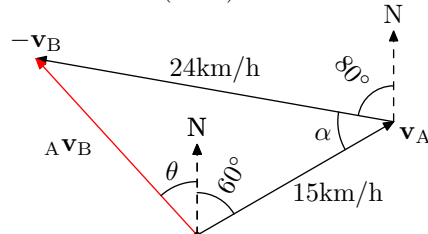
By Pythagoras,

$$\begin{aligned} |{}_A\mathbf{v}_B|^2 &= 6^2 + 8^2 \\ |{}_A\mathbf{v}_B| &= 10\text{km/h} \end{aligned}$$

$$\begin{aligned}\tan(90^\circ - \theta) &= \frac{8}{6} \\ 90^\circ - \theta &\approx 53.13^\circ \\ \theta &\approx 36.87^\circ\end{aligned}$$

${}_A\mathbf{v}_B = 10\text{km/h}$  on a bearing of  $037^\circ$ .

7.  ${}_A\mathbf{v}_B = \mathbf{v}_A + (-\mathbf{v}_B)$



$$\begin{aligned} 60 + (80 + \alpha) &= 180 \text{ (cointerior angles)} \\ \alpha &= 40^\circ \end{aligned}$$

By the cosine rule,

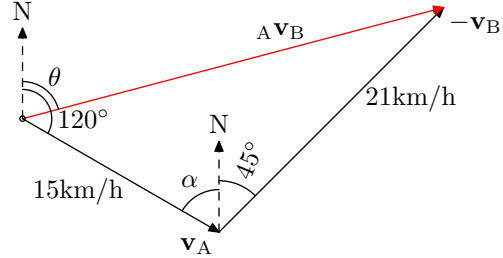
$$\begin{aligned} |{}_A\mathbf{v}_B|^2 &= 24^2 + 15^2 - 2 \times 24 \times 15 \cos 40^\circ \\ |{}_A\mathbf{v}_B| &\approx 15.79\text{km/h} \end{aligned}$$

By the cosine rule again (since the sine rule would be ambiguous here),

$$\begin{aligned}\theta + 60 &= \cos^{-1} \frac{15^2 + 15.79^2 - 24^2}{2 \times 15 \times 15.79} \\ &= 102.38^\circ \\ \theta &= 42.38^\circ\end{aligned}$$

${}_A\mathbf{v}_B = 15.8\text{km/h}$  on a bearing of  $318^\circ$ .

8.  ${}_A\mathbf{v}_B = \mathbf{v}_A + (-\mathbf{v}_B)$



$$\begin{aligned}\alpha &= 180 - 120 \text{ (cointerior angles)} \\ &= 60^\circ\end{aligned}$$

$$\alpha + 45 = 105^\circ$$

By the cosine rule,

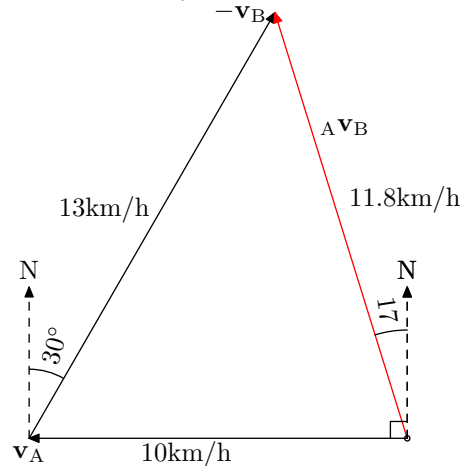
$$\begin{aligned} |{}_A\mathbf{v}_B|^2 &= 21^2 + 15^2 - 2 \times 21 \times 15 \cos 105^\circ \\ |{}_A\mathbf{v}_B| &= 28.79\text{km/h} \end{aligned}$$

By the sine rule (unambiguous here),

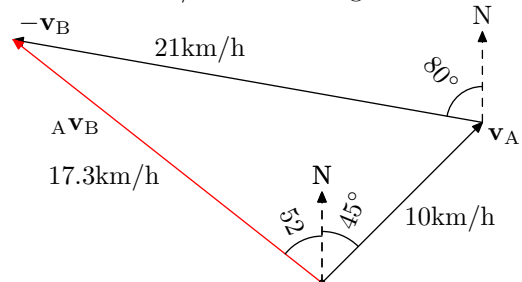
$$\begin{aligned}\frac{\sin(120^\circ - \theta)}{21} &= \frac{\sin 105^\circ}{28.79} \\ 120^\circ - \theta &= \sin^{-1} \frac{21 \sin 105^\circ}{28.79} \\ &= 44.79^\circ \\ \theta &= 120 - 44.79 \\ &= 75.21^\circ\end{aligned}$$

${}_A\mathbf{v}_B = 28.8\text{km/h}$  on a bearing of  $075^\circ$ .

9.  ${}_A\mathbf{v}_B = 11.8\text{km/h}$  on a bearing of  $343^\circ$ .



10.  ${}_A\mathbf{v}_B = 17.3\text{km/h}$  on a bearing of  $308^\circ$ .

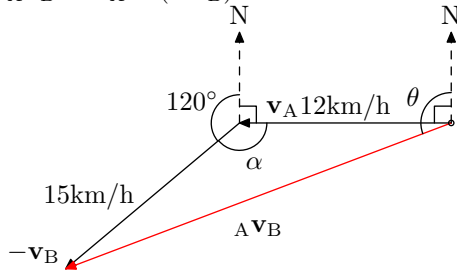


11.  $\mathbf{v}_A = 7\mathbf{i} - 10\mathbf{j}$   
 $\mathbf{v}_B = 2\mathbf{j} + 20\mathbf{j}$

(a)  ${}_A\mathbf{v}_B = \mathbf{v}_A - \mathbf{v}_B = 5\mathbf{i} - 30\mathbf{j}\text{km/h}$

(b)  ${}_B\mathbf{v}_A = \mathbf{v}_B - \mathbf{v}_A = -5\mathbf{i} + 30\mathbf{j}\text{km/h}$

12. (a)  ${}_A\mathbf{v}_B = \mathbf{v}_A + (-\mathbf{v}_B)$



$$\begin{aligned}\alpha &= 360 - 90 - 120 \\ &= 150^\circ\end{aligned}$$

By the cosine rule,

$$\begin{aligned}|\mathbf{A v}_B|^2 &= 12^2 + 15^2 - 2 \times 12 \times 15 \cos 150^\circ \\ |\mathbf{A v}_B| &= 26.09\text{km/h}\end{aligned}$$

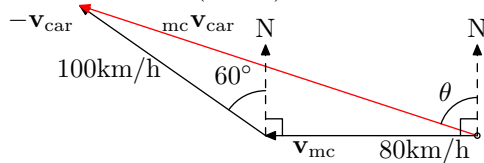
By the sine rule (unambiguous here),

$$\begin{aligned}\frac{\sin(\theta - 90)}{15} &= \frac{\sin 150^\circ}{26.09} \\ \theta - 90 &= \sin^{-1} \frac{15 \sin 150^\circ}{26.09} \\ &= 16.71^\circ \\ \theta &= 106.71^\circ\end{aligned}$$

$\mathbf{A v}_B = 26.1\text{km/h}$  on a bearing of  $253^\circ$ .

(b)  ${}_B\mathbf{v}_A = \mathbf{v}_B + (-\mathbf{v}_A) = -\mathbf{A v}_B$   
(Same diagram with all the arrows reversed.)  
 ${}_B\mathbf{v}_A = 26.1\text{km/h}$  on a bearing of  $073^\circ$ .

13.  ${}_{mc}\mathbf{v}_{car} = \mathbf{v}_{mc} + (-\mathbf{v}_{car})$



By the cosine rule,

$$\begin{aligned}|\mathbf{mc v}_{car}|^2 &= 100^2 + 80^2 - 2 \times 100 \times 80 \cos 150^\circ \\ |\mathbf{mc v}_{car}| &= 173.94\text{km/h}\end{aligned}$$

By the sine rule (unambiguous here),

$$\begin{aligned}\frac{\sin(90 - \theta)}{100} &= \frac{\sin 150^\circ}{173.94} \\ 90 - \theta &= \sin^{-1} \frac{100 \sin 150^\circ}{173.94} \\ &= 16.91^\circ \\ \theta &= 73.29^\circ\end{aligned}$$

${}_{mc}\mathbf{v}_{car} = 174\text{km/h}$  on a bearing of  $287^\circ$ .

14.  ${}_A\mathbf{v}_B = \mathbf{v}_A - \mathbf{v}_B$

$\mathbf{v}_B = \mathbf{v}_A - \mathbf{A v}_B = \mathbf{i} - 12\mathbf{j}$

15.  ${}_B\mathbf{v}_A = \mathbf{v}_B - \mathbf{v}_A$

$\mathbf{v}_B = \mathbf{v}_A + {}_B\mathbf{v}_A = -2\mathbf{i} + 6\mathbf{j}$

16. LHS:

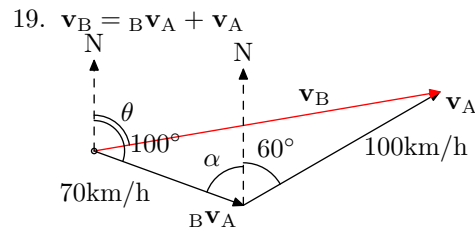
$$\begin{aligned}{}_A\mathbf{v}_B + {}_B\mathbf{v}_C &= (\mathbf{v}_A - \mathbf{v}_B) + (\mathbf{v}_B - \mathbf{v}_C) \\ &= \mathbf{v}_A - \mathbf{v}_B + \mathbf{v}_B - \mathbf{v}_C \\ &= \mathbf{v}_A - \mathbf{v}_C \\ &= {}_A\mathbf{r}_C \\ &= \text{RHS. Q.E.D.}\end{aligned}$$

17.  ${}_A\mathbf{v}_B = \mathbf{v}_A - \mathbf{v}_B$

$$\begin{aligned}\mathbf{v}_B &= \mathbf{v}_A - \mathbf{A v}_B \\ &= 20 - 30 \\ &= -10\text{km/h due North} \\ &= 10\text{km/h due South}\end{aligned}$$

18.  ${}_A\mathbf{v}_B = \mathbf{v}_A - \mathbf{v}_B$

$$\begin{aligned}\mathbf{v}_B &= \mathbf{v}_A - \mathbf{A v}_B \\ &= 80 - 60 \\ &= 20\text{km/h due South}\end{aligned}$$



19.  $\mathbf{v}_B = {}_B\mathbf{v}_A + \mathbf{v}_A$

$$\begin{aligned}\alpha &= 180 - 100 \text{ (cointerior angles)} \\ &= 80^\circ \\ \alpha + 60 &= 140^\circ\end{aligned}$$

By the cosine rule,

$$\begin{aligned}|\mathbf{v}_B|^2 &= 70^2 + 100^2 - 2 \times 70 \times 100 \cos 140^\circ \\ |\mathbf{A v}_B| &= 160.08\text{km/h}\end{aligned}$$

By the sine rule (unambiguous here),

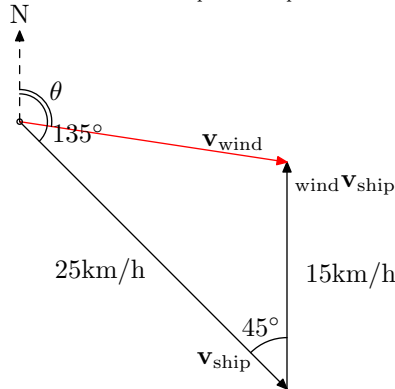
$$\begin{aligned}\frac{\sin(100^\circ - \theta)}{100} &= \frac{\sin 140^\circ}{160.08} \\ 100^\circ - \theta &= \sin^{-1} \frac{100 \sin 140^\circ}{160.08} \\ &= 23.68^\circ \\ \theta &= 100 - 23.68 \\ &= 76.32^\circ\end{aligned}$$

$\mathbf{v}_B = 160\text{km/h}$  on a bearing of  $076^\circ$ .

20.  $\mathbf{v}_{wind} = \text{wind}\mathbf{v}_{ship} + \mathbf{v}_{ship} = (13\mathbf{i} + \mathbf{j})\text{km/h}$

21.  $\mathbf{v}_{wind} = \text{wind}\mathbf{v}_{walker} + \mathbf{v}_{walker} = (4\mathbf{i} - 2\mathbf{j})\text{km/h}$

22.  $\mathbf{v}_{\text{wind}} = \text{wind} \mathbf{v}_{\text{ship}} + \mathbf{v}_{\text{ship}}$



By the cosine rule,

$$|\mathbf{v}_{\text{wind}}|^2 = 25^2 + 15^2 - 2 \times 25 \times 15 \cos 45^\circ$$

$$|\mathbf{v}_{\text{wind}}| = 17.88 \text{ km/h}$$

By the sine rule (unambiguous here),

$$\frac{\sin(135^\circ - \theta)}{15} = \frac{\sin 45^\circ}{17.88}$$

$$135^\circ - \theta = \sin^{-1} \frac{15 \sin 45^\circ}{17.88}$$

$$= 36.39^\circ$$

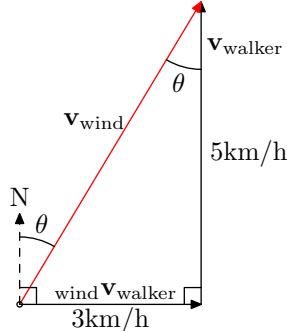
$$\theta = 135 - 36.39$$

$$= 98.61^\circ$$

$\mathbf{v}_{\text{wind}} = 17.9 \text{ km/h}$  on a bearing of  $099^\circ$ . However, it is usual to give the direction a wind blows *from* so:

$\mathbf{v}_{\text{wind}} = 17.9 \text{ km/h}$  blowing from  $279^\circ$ .

23.  $\mathbf{v}_{\text{wind}} = \text{wind} \mathbf{v}_{\text{walker}} + \mathbf{v}_{\text{walker}}$



By Pythagoras' Theorem,

$$|\mathbf{v}_{\text{wind}}|^2 = 5^2 + 3^2$$

$$|\mathbf{v}_{\text{wind}}| = \sqrt{34} \text{ km/h}$$

$$\approx 5.83 \text{ km/h}$$

$$\tan \theta = \frac{3}{5}$$

$$\theta = 30.96^\circ$$

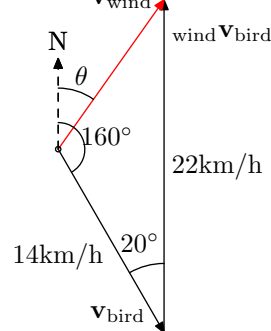
$\mathbf{v}_{\text{wind}} = 5.8 \text{ km/h}$  on a bearing of  $031^\circ$ . However, it is usual to give the direction a wind blows *from* so:

$\mathbf{v}_{\text{wind}} = 5.8 \text{ km/h}$  blowing from  $211^\circ$ .

24. • B appears stationary, so  $\mathbf{v}_B = \mathbf{v}_A$  and B is travelling due north at  $10 \text{ km/h}$ .

- C appears to be moving south at  $3 \text{ km/h}$ . This means that C is actually moving north  $3 \text{ km/h}$  slower than A. C is travelling due north at  $7 \text{ km/h}$ .
- D appears to be moving north at  $5 \text{ km/h}$ . It is actually travelling north  $5 \text{ km/h}$  faster than A. D is travelling due north at  $15 \text{ km/h}$ .

25.  $\mathbf{v}_{\text{wind}} = \text{wind} \mathbf{v}_{\text{bird}} + \mathbf{v}_{\text{bird}}$



By the cosine rule,

$$|\mathbf{v}_{\text{wind}}|^2 = 14^2 + 22^2 - 2 \times 14 \times 22 \cos 20^\circ$$

$$|\mathbf{v}_{\text{wind}}| = 10.06 \text{ km/h}$$

Without drawing a scale diagram, we don't know whether the angle  $160^\circ - \theta$  is acute or obtuse, so the sine rule is ambiguous here. We'll use the cosine rule instead.

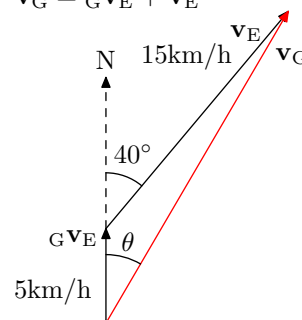
$$160^\circ - \theta = \cos^{-1} \frac{14^2 + 10.06^2 - 22^2}{2 \times 14 \times 10.06}$$

$$= 131.57^\circ$$

$$\theta = 28.43^\circ$$

$\mathbf{v}_{\text{wind}} = 10.1 \text{ km/h}$  from  $208^\circ$ .

26. • Ship F appears to be moving at the same speed as ship E but in the opposite direction ( $220^\circ - 20^\circ = 180^\circ$ ). This implies that ship F is actually stationary.
- $\mathbf{v}_G = \text{G} \mathbf{v}_E + \mathbf{v}_E$



$$|\mathbf{v}_G|^2 = 5^2 + 15^2 - 2 \times 5 \times 15 \cos 140^\circ$$

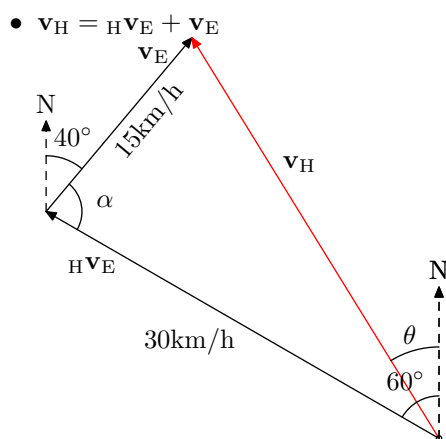
$$|\mathbf{v}_G| = 19.10 \text{ km/h}$$

$$\frac{\sin(\theta)}{15} = \frac{\sin 140^\circ}{19.10}$$

$$\theta = \sin^{-1} \frac{15 \sin 140^\circ}{19.10}$$

$$= 30.31^\circ$$

$\mathbf{v}_G = 19.1 \text{ km/h}$  on a bearing of  $030^\circ$ .



$$40 + \alpha = 180 - 60 \text{ (cointerior angles)}$$

$$\alpha = 80^\circ$$

$$|\mathbf{v}_H|^2 = 30^2 + 15^2 - 2 \times 30 \times 15 \cos 80^\circ$$

$$|\mathbf{v}_H| = 31.12 \text{ km/h}$$

$$\frac{\sin(60^\circ - \theta)}{15} = \frac{\sin 80^\circ}{31.12}$$

$$60 - \theta = \sin^{-1} \frac{15 \sin 80^\circ}{31.12}$$

$$= 28.33^\circ$$

$$\theta = 60 - 28.33$$

$$= 31.67^\circ$$

$\mathbf{v}_H = 31.1 \text{ km/h}$  on a bearing of  $328^\circ$ .

27.

$${}_A\mathbf{v}_B = 7\mathbf{i} - 10\mathbf{j}$$

$${}_A\mathbf{v}_C = 13\mathbf{i} - 2\mathbf{j}$$

$${}_B\mathbf{v}_C = \mathbf{v}_B - \mathbf{v}_C$$

$$= \mathbf{v}_B - \mathbf{v}_C + \mathbf{v}_A - \mathbf{v}_A$$

$$= \mathbf{v}_B - \mathbf{v}_A + \mathbf{v}_A - \mathbf{v}_C$$

$$= {}_B\mathbf{v}_A + {}_A\mathbf{v}_C \text{ (we might have started here)}$$

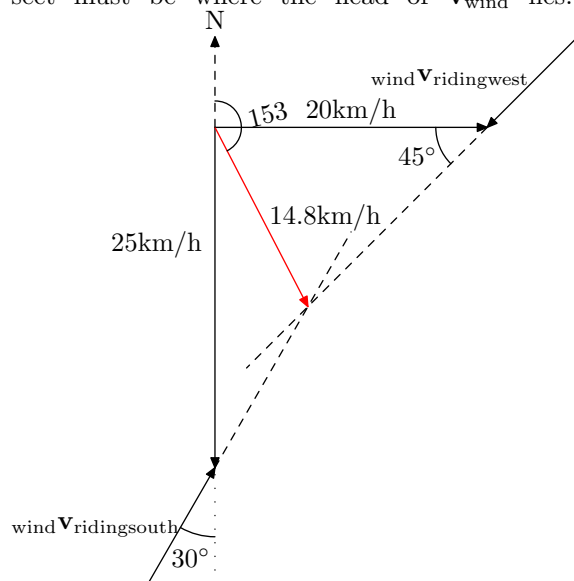
$$= -(7\mathbf{i} - 10\mathbf{j}) + (13\mathbf{i} - 2\mathbf{j})$$

$$= 6\mathbf{i} + 8\mathbf{j}$$

28. On the same diagram, draw the vectors representing the speed and direction of the cyclist travelling east and travelling south. From the head of each of these vectors draw the line representing the apparent direction of the wind. Since

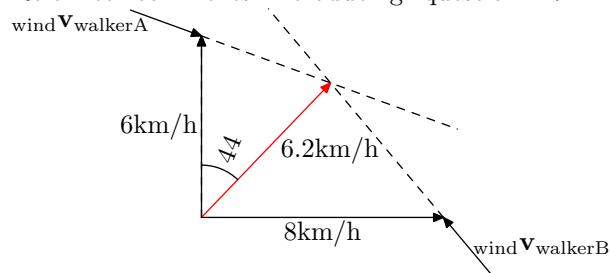
$$\mathbf{v}_{\text{wind}} = \mathbf{v}_{\text{cyclist}} + \text{wind} \mathbf{v}_{\text{cyclist}}$$

with its foot at the origin, the head of  $\mathbf{v}_{\text{wind}}$  must lie along this line. Thus, the point where these two extended lines intersect must be where the head of  $\mathbf{v}_{\text{wind}}$  lies.



The wind velocity is  $14.8 \text{ km/h}$  from  $333^\circ$ .

29. Refer to comments introducing question 27.



The wind velocity is  $6.2 \text{ km/h}$  from  $224^\circ$ .

## Miscellaneous Exercise 7

1. (a)

$$\begin{aligned}
 AC &= \sqrt{(AB)^2 + (BC)^2} \\
 &= \sqrt{162^2 + 94^2} \\
 &= \sqrt{35080} \\
 &= 187.3\text{mm}
 \end{aligned}$$

(b)

$$\begin{aligned}
 AE &= \sqrt{(AF)^2 + (EF)^2} \\
 &= \sqrt{\left(\frac{AC}{2}\right)^2 + (EF)^2} \\
 &= \sqrt{\frac{35080}{4} + 118^2} \\
 &= \sqrt{22694} \\
 &= 150.6\text{mm}
 \end{aligned}$$

(c) Let  $\theta$  be the angle face ABE makes with the base ABCD.

Let point G be the midpoint of AB.

$$\begin{aligned}
 \tan \theta &= \frac{EF}{FG} \\
 \angle EBF &= \tan^{-1} \frac{118}{94/2} \\
 &= 68.3^\circ
 \end{aligned}$$

2. (a) Area =  $\frac{1}{2}r^2\theta = .5 \times 5^2 \times 2 = 25\text{cm}^2$ (b) Area =  $\frac{1}{2}r^2(\theta - \sin \theta) = .5 \times 8^2(1.8 - \sin 1.8) = 26.4\text{cm}^2$ 

(c)

$$\begin{aligned}
 \sin \frac{\theta}{2} &= \frac{8}{10} \\
 \theta &= 2 \sin^{-1} 0.8 \\
 &= 1.85
 \end{aligned}$$

For the major segment we want the reflex angle:

$$\alpha = 2\pi - \theta = 4.43$$

$$\begin{aligned}
 \text{Area} &= \frac{1}{2}r^2(\alpha - \sin \alpha) \\
 &= .5 \times 10^2(4.43 - \sin 4.43) \\
 &= 269.4\text{cm}^2
 \end{aligned}$$

3. (a) For  $x < 3$ ,  $|2x - 6| = -(2x - 6)$ 

$$\begin{aligned}
 -(2x - 6) &\leq x \\
 -2x + 6 &\leq x \\
 6 &\leq 3x \\
 2 &\leq x \\
 x &\geq 2
 \end{aligned}$$

For  $x \geq 3$ ,  $|2x - 6| = 2x - 6$ 

$$\begin{aligned}
 2x - 6 &\leq x \\
 x - 6 &\leq 0 \\
 x &\leq 6
 \end{aligned}$$

Solution:  $2 \leq x \leq 6$ (b) For  $x < 0$ ,  $|x| = -x$ 

$$\begin{aligned}
 3 - 2x &> -x \\
 3 &> x \\
 x &< 3
 \end{aligned}$$

So the inequality is satisfied for all  $x < 0$ .For  $x \geq 0$ ,  $|x| = x$ 

$$\begin{aligned}
 3 - 2x &> x \\
 3 &> 3x \\
 x &< 1
 \end{aligned}$$

Solution:  $x < 1$ (c) For  $x < -3$ ,  $|x + 3| = -(x + 3)$ 

$$\begin{aligned}
 -(x + 3) &< x + 1 \\
 -x - 3 &< x + 1 \\
 -4 &< 2x \\
 -2 &< x \\
 x &> -2
 \end{aligned}$$

No solution for  $x < -3$ . For  $x \geq -3$ ,  $|x + 3| = x + 3$ 

$$\begin{aligned}
 x + 3 &< x + 1 \\
 3 &< 1
 \end{aligned}$$

No solution for  $x \geq -3$ .No value of  $x$  satisfies the inequality.(d) For  $x > -1$  this problem is essentially the same as the previous one which has no solution, so we only need to consider  $x \leq -1$  and the problem becomes

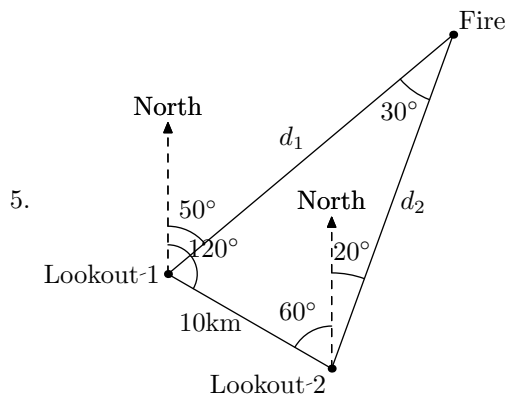
$$\begin{aligned}
 |x + 3| &\leq -(x + 1) \\
 \text{For } x < -3, |x + 3| &= -(x + 3) \\
 -(x + 3) &\leq -(x + 1) \\
 -x - 3 &\leq -x - 1 \\
 -3 &\leq -1
 \end{aligned}$$

The inequality is satisfied for all  $x < -3$ .For  $-3 \leq x \leq -1$ ,  $|x + 3| = x + 3$ 

$$\begin{aligned}
 x + 3 &\leq -(x + 1) \\
 x + 3 &\leq -x - 1 \\
 2x &\leq -4 \\
 x &\leq -2
 \end{aligned}$$

Solution:  $x \leq -2$

4. (a)  $d = r\theta = \frac{2\pi}{5} \times 1.8 = 2.26\text{m}$   
 (b)  $d = r\theta = \frac{2\pi}{5} \times 1 = 1.26\text{m}$



$$\frac{d_1}{\sin 80^\circ} = \frac{10}{\sin 30^\circ}$$

$$d_1 = \frac{10 \sin 80^\circ}{\sin 30^\circ}$$

$$= 19.7\text{km}$$

$$\frac{d_2}{\sin 70^\circ} = \frac{10}{\sin 30^\circ}$$

$$d_2 = \frac{10 \sin 70^\circ}{\sin 30^\circ}$$

$$= 18.8\text{km}$$

6.

$$\mathbf{a} = k\mathbf{b}$$

$$2\mathbf{i} + 2\mathbf{j} = k(x\mathbf{i} + 5\mathbf{j})$$

$$x = 5$$

$$|\mathbf{b}| = |\mathbf{c}|$$

$$|5\mathbf{i} + 5\mathbf{j}| = |7\mathbf{i} + y\mathbf{j}|$$

$$\sqrt{5^2 + 5^2} = \sqrt{7^2 + y^2}$$

$$50 = 49 + y^2$$

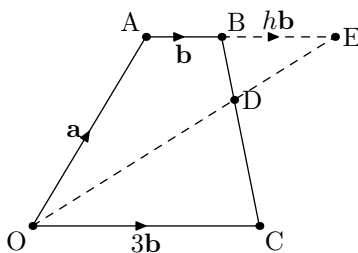
$$y = 1$$

7. (a)  $\overrightarrow{BC} = -\mathbf{b} - \mathbf{a} + 3\mathbf{b} = 2\mathbf{b} - \mathbf{a}$   
 (b)  $\overrightarrow{BD} = \frac{1}{3}\overrightarrow{BC} = \frac{2}{3}\mathbf{b} - \frac{1}{3}\mathbf{a}$   
 (c)

$$\overrightarrow{OD} = \mathbf{a} + \mathbf{b} + \overrightarrow{BD}$$

$$= \mathbf{a} + \mathbf{b} + \frac{2}{3}\mathbf{b} - \frac{1}{3}\mathbf{a}$$

$$= \frac{2}{3}\mathbf{a} + \frac{5}{3}\mathbf{b}$$



$$\overrightarrow{OE} = \overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{BD}$$

$$= \mathbf{a} + \mathbf{b} + h\mathbf{b}$$

$$= \mathbf{a} + (h+1)\mathbf{b}$$

$$\overrightarrow{OE} = \overrightarrow{OD} + \overrightarrow{DE}$$

$$= (k+1)\overrightarrow{OD}$$

$$= (k+1)\left(\frac{2}{3}\mathbf{a} + \frac{5}{3}\mathbf{b}\right)$$

$$\therefore \mathbf{a} + (h+1)\mathbf{b} = (k+1)\left(\frac{2}{3}\mathbf{a} + \frac{5}{3}\mathbf{b}\right)$$

$$\mathbf{a} - \frac{2(k+1)}{3}\mathbf{a} = \frac{5(k+1)}{3}\mathbf{b} - (h+1)\mathbf{b}$$

$$\left(1 - \frac{2k}{3} - \frac{2}{3}\right)\mathbf{a} = \left(\frac{5k}{3} + \frac{5}{3} - h - 1\right)\mathbf{b}$$

$$\text{LHS: } \frac{1}{3} - \frac{2k}{3} = 0$$

$$k = \frac{1}{2}$$

$$\text{RHS: } \frac{5k}{3} + \frac{2}{3} - h = 0$$

$$\frac{5}{6} + \frac{2}{3} - h = 0$$

$$\frac{9}{6} - h = 0$$

$$h = \frac{3}{2}$$

8. (a)  $\log_a x + \log_a y = \log_a xy$  so  $p = xy$

(b)  $p = x^y$

(c)

$$3\log_a x - \log_a y = \log_a p$$

$$\log_a x^3 - \log_a y = \log_a p$$

$$\log_a \frac{x^3}{y} = \log_a p$$

$$p = \frac{x^3}{y}$$

(d)

$$2 + .5\log_{10} y = \log_{10} p$$

$$\log_{10} 10^2 + .5\log_{10} y = \log_{10} p$$

$$\log_{10} 100 + \log_{10} y^{0.5} = \log_{10} p$$

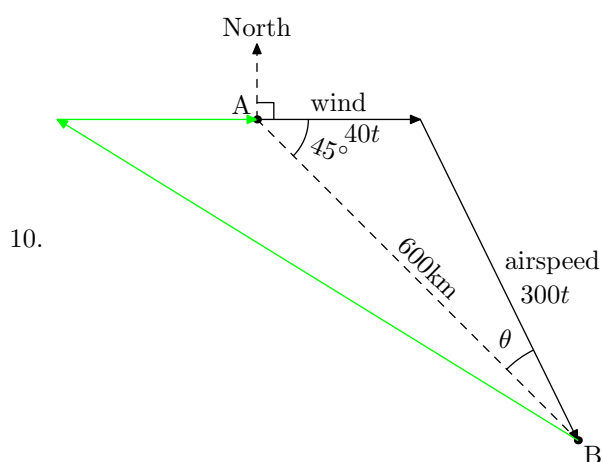
$$\log_{10} 100 + \log_{10} \sqrt{y} = \log_{10} p$$

$$\log_{10}(100\sqrt{y}) = \log_{10} p$$

$$p = 100\sqrt{y}$$

9. Let point P be the position of the object after 2 seconds.

$$\begin{aligned}
 {}_P\mathbf{r}_B &= 2(3\mathbf{i} - 2\mathbf{j}) \\
 &= 6\mathbf{i} - 4\mathbf{j} \\
 {}_P\mathbf{r}_A &= {}_P\mathbf{r}_B + {}_B\mathbf{r}_A \\
 &= 6\mathbf{i} - 4\mathbf{j} + 8\mathbf{i} + 3\mathbf{j} \\
 &= 14\mathbf{i} - \mathbf{j} \\
 \mathbf{r}_P &= {}_P\mathbf{r}_A + \mathbf{r}_A \\
 &= 14\mathbf{i} - \mathbf{j} + -2\mathbf{i} + 7\mathbf{j} \\
 &= 12\mathbf{i} + 6\mathbf{j} \\
 |\mathbf{r}_P| &= |12\mathbf{i} + 6\mathbf{j}| \\
 &= 6|2\mathbf{i} + \mathbf{j}| \\
 &= 6\sqrt{2^2 + 1^2} \\
 &= 6\sqrt{5} \text{ m from the origin.}
 \end{aligned}$$



Flying from A to B:

$$\begin{aligned}
 (300t)^2 &= (40t)^2 + 600^2 - 2 \times 40t \times 600 \cos 45^\circ \\
 90000t^2 &= 1600t^2 + 360000 - 48000t \times \frac{\sqrt{2}}{2} \\
 225t^2 &= 4t^2 + 900 - 120t \times \frac{\sqrt{2}}{2} \\
 0 &= 221t^2 + 60\sqrt{2}t - 900 \\
 t &= 1.835 \text{ hours} \\
 t &= 110 \text{ minutes}
 \end{aligned}$$

Flying from B to A:

$$\begin{aligned}
 (300t)^2 &= (40t)^2 + 600^2 - 2 \times 40t \times 600 \cos 135^\circ \\
 90000t^2 &= 1600t^2 + 360000 + 48000t \times \frac{\sqrt{2}}{2} \\
 225t^2 &= 4t^2 + 900 + 120t \times \frac{\sqrt{2}}{2} \\
 0 &= 221t^2 - 60\sqrt{2}t - 900 \\
 t &= 2.219 \text{ hours} \\
 t &= 133 \text{ minutes}
 \end{aligned}$$